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OPTIMAL DECISION MAKING FOR WELL INTERVENTIONS UNDER UNCERTAINTY

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Abstract. Decision making in real-world problems takes place in an environment of imprecise probabilities. As a result, the decision theory based on classical probability is not an adequate tool for such cases. In this paper we consider the problem of decision making for carrying out of geological and engineering operations in oil extraction when relevant information is described by interval-valued probabilities. Decision making is based on the use of utility function described as Choquet integral with a lower probability as a non-additive measure. The obtained results show applicability of the proposed approach.

Keywords: imprecise probability, non-additive measure, Choquet integral, utility function, oil well.

AMS Subject Classification: 60A10, 94D05, 94C30.

1. Introduction

Optimization of oil field development performances and producing well operational conditions are one of the key issues in the oil extraction processes. For this purpose, various measures are taken at all stages of field development, particularly at the final stage. All additional measures taken at this stage are focused on the increase in overall oil production and therefore improvement of engineering and economic performances. For this purpose, carrying out of geological and engineering operations in parallel with routine work is of special importance.

One of the most advanced methods of regulation of producing well technological process parameters and optimization of their flow rate is the injection of polymeric solutions into the space behind the well bore [11]. Injection of polymeric solutions into the well reduces hydraulic losses in the well bore, facilitates flowing of mixtures (oil, gas, water) entering from strata through the bore, creates favorable conditions for normal operation of the lift and as a result promotes increase in the well productivity.

However implementation of any operation and obtaining positive results in the oil extraction practice depend on the right choice of a well. Thus, first of all, well's condition should be normal and meet appropriate requirements for implementation of any operations, otherwise, effectiveness of such operations will be inadequate. Thus, the problem consists in determination of the number of producing wells in the oil production enterprise's stock for which well intervention will be carried out, i. e. the total number of wells. As experience shows, it is impossible to obtain high efficiency for all wells in the result of intervention. In certain cases a decrease in capacity

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of 10-15 % of wells under intervention is observed. In this context, it is important to make the right decision on carrying out of intervention and to determine an optimal number of wells.

One of the main factors that complicates decision making on a choice of an optimal number of wells is imprecision of relevant information. Indeed, information on amount of oil production and related probabilities is naturally imprecise. In such cases, the use of the classical probability theory based approaches of decision making [7, 17] is inadequate. The issue is that in decision making with imprecise probabilities, human beliefs exhibit non-additivity property. Decision making with imprecise probabilities and non-additive beliefs constitute nowadays a wide area of research aimed at dealing with real-world problems [6, 8, 12, 25]. In this paper we consider a problem of decision making on a choice of an optimal number of oil wells when decision relevant information is described by interval-valued outcomes and probabilities. For solving the considered decision making problem, an interval-valued Choquet integral based utility function is used. The paper is structured as follows. In Section 2 we provide necessary prerequisite material to be used in the study. In Section 3 we formulate a problem of decision making with interval-valued information. Section 4 is devoted to the decision making approach based on the use of a utility function described as an interval-valued Choquet integral. A real world application of the approach to decision making on a choice of an optimal number of oil wells is proposed in Section 5. Section 6 is conclusion.

2. Preliminaries

Denote $S = \{S_1, ..., S_n\}$ a set of states of nature in a decision making problem and FS a σ-algebra of subsets of S.

Definition 2.1. ***Interval probability [12].*** *The intervals* $P(S_i) = \left[\underline{p_i}, \overline{p_i}\right]$ $i = 1, \ldots, n$ *are called the interval probabilities of* S *if for any* $p_i \in \left[\underline{p_i}, \overline{p_i}\right]$ *there exist* $p_1 \in \left[\underline{p_1}, \overline{p_1}\right], \ldots,$ $p_{i-1} \in \left[\underline{p}_{i-1}, \overline{p}_{i-1}\right]$, $p_{i+1} \in \left[\underline{p}_{i+1}, \overline{p}_{i+1}\right], \ldots p_n \in \left[\underline{p}_n, \overline{p}_n\right]$ *such that* $\sum_{i=1}^{n} p_i = 1$

In this definition p_i *denotes a basic probability, i. e. a numeric probability from an interval* $P(S_i) = \left[\underline{p_i}, \overline{p_i}\right]$.

From Definition 1 it follows that in contrast to numerical probabilities, interval probabilities cannot be directly assigned. The issue is that the requirement to numerical probabilities to sum up to one must be satisfied throughout all the probability intervals. Sometimes, interval probabilities $P(S_i) = P_i$ can be directly assigned to $n-1$ states of nature $S_1, S_2, ..., S_{j-1}, S_{j+1}, ..., S_n$. Then an interval probability $P(S_j) = P_j$ for the rest one state of nature S_j will be calculated on the basis of these probabilities.

Definition 2.2. ***Interval probability of a subset of*** $S = \{S_1, ..., S_n\}$ ***[12].*** *Let* $S = \{S_1, ..., S_n\}$ *be the states of nature and let the interval probabilities* $P(S_i) = \left[\underline{p_i}, \bar{p}_i\right]$ $i = 1, \ldots, n$ *of the elements of* S *be given. The interval probability of a subset* H *of* S *is* $P(H) = \left[\underline{p}, \bar{p}\right]$ *where the lower and upper bounds are defined as follows:*

$$\underline{p} = \sum_{j,\, S_j \in H} p_j \to \min,$$

subject to

$$\underline{p}_j \le p_j \le \bar{p}_j,$$
$$p_1 + ... + p_n = 1$$

and

$$\bar{p} = \sum_{j,\, S_j \in H} p_j \to \max,$$

subject to

$$\underline{p}_j \le p_j \le \bar{p}_j,$$
$$p_1 + ... + p_n = 1,$$

where $p_j \in P(S_j)$, $P(S_j) = \left[\underline{p}_j, \bar{p}_j\right]$.

Definition 2.3. ***Non-additive measure*** *[22]. A set function* $\eta : F \to [0,1]$ *is referred to as a non-additive measure if it satisfies the following:*

(1) $\eta\ (\emptyset) = 0$
(2) $\forall H, G \in F,\quad H \subset G$, *implies* $\eta(H) \le \eta(G)$
(3) $\eta\ (S) = 1$

Definition 2.4. ***Choquet Expected Utility (CEU)****. Choquet Expected Utility [22] is a utility model which is able to describe non-additivity of preferences. One of the main reasons of this non-additivity is imprecise information on probabilities. A value of utility of alternative f in CEU model for finite set of states of nature* $S = \{S_1, ..., S_n\}$ *CEU is described as follows:*

$$U(f) = \sum_{i=1}^{n} \left(u\ \left(f\left(S_{(i)}\right)\right) - u\ \left(f\left(S_{(i+1)}\right)\right)\right)\ \eta\left(\left\{S_{(1)}, ..., S_{(i)}\right\}\right),$$

where i in the index of the states S implies that they are permuted such that $u\left(f\left(S_{(i)}\right)\right) \ge u\left(f\left(S_{(i+1)}\right)\right)$, *and* $u\left(f\left(S_{(n+1)}\right)\right) = 0$ *by convention.*

3. Statement of the problem

The primary objective of carrying out of well interventions in oil extraction processes is to obtain high revenue by increasing oil production. The basis of this process management consists in oil production increase maximization (and accordingly increase in revenue) and investment risk minimization [13].

Effectiveness of the methods applicable for improving well operation conditions to a large extent depends on accurate determination of the total number of wells where research will be conducted. In addition, although long-term operation of wells is impossible due to carrying out of various operations, no general procedure was developed for current operations effectiveness estimation. In such conditions, determination of appropriate volume of intervention in the producing well stock has come to the fore as the major practical issue. In addition, currently there are practically no criteria for determination of an optimal number of wells which would take into account engineering and economic parameters for taking various well productivity optimization measures.

Thus, a need arises in decision making on determination of an optimal number of wells to improve wells productivity. The considered problem is a problem of decision making under uncertainty characterized by imprecise decision relevant information. More concretely, for each available option (alternative number of wells) possible outcomes (oil production) and their probabilities are naturally imprecise. Thus, we will consider determination of an optimal number of well in case of the highest increase in well flow rate [16] as a problem of decision making with interval-valued information.

The considered problem of decision making can be formalized as 4-tuple (S, P, A, X, $\succeq$) [2, 22]: $S = \{S_1, ..., S_n\}$ is a space of mutually exclusive and exhaustive states of nature that describe possible objective conditions which determine oil production; P is a distribution of interval probabilities of states of nature, $P(S_i)$, $i = 1,\ldots,$ n, $S_i \in S$; X is a set of interval-valued outcomes (oil production); A is a set of alternatives (alternative number of wells) that are F_S - measurable functions $f : S \to X$ where F_S is a σ-algebra of subsets of S, $B \subset S$. $\preceq$ is a non-additive preference relation over A which satisfy the properties of weak order, transitivity, continuity, comonotonic independence, monotonicity and non-degeneracy [2, 22].

The problem is to determine preferences $\succeq$ among alternatives A.

4. Solution of the problem

Today the theory of decision making is described by various mathematical methods. One of the most important parts of the theory of decision making is the utility theory. The utility models based on the Choquet integral are among the most effective utility functions [15]. It is more adequate for modeling decision-maker (DM) behavior because it is based on the use of non-additive measures and not on probability measures [5, 9, 10, 14, 18, 19, 20, 23, 26]. The model can reflect a number of typical properties associated with risk and uncertainty attitude of a DM. Alternatively, most available utility models assume that the utility function and probability distribution are precisely known. Studies [1, 2, 3, 4] suggest new decision models which allow to solve decision making problems characterized by imprecise decision relevant information. These models are based on a fuzzy valued Choquet integral. We will apply this model to the considered decision making problem characterized by interval-valued information. The interval-valued Choquet integral based utility of an alternative $f_i \in A$ is described as follows:

$$U(f_i) = \sum_{j=1}^{m} \left(X_{i\ (j)} - X_{i\ (j+1)}\right)\ \eta_{\{(1),\ldots,(j)\}},$$

where $\eta_{\{(1),\ldots,(j)\}}$ is a non-additive measure, $X_{i\ (j)} = f_i\left(S_{(j)}\right)$ is an interval-valued outcome of alternative fi at a state $S_{(j)}$, () index means that $X_{i\ (j)}$ values are enumerated such that, $X_{i\ (j)} \geq X_{i\ (j+1)}$ and $X_{i\ (m+1)} = 0$ by convention. By means of this measure non-additivity of DM's preferences under ambiguity (e. g. a safe to a certain extent decision or a risky to a certain extent decision made under uncertainty) can be modeled.

Let us assume that a DM wants to make a safe choice under interval valued information on probabilities $P(S_i) = \left[\underline{p}_i, \bar{p}_i\right]$. Then, a lower probability can be used as a non-additive measure to describe a DM's beliefs [24]. In other words, we assume that a DM thinks about the worst possible case of oil production events within the probability ranges. The lower probability η is defined as follows:

$$\eta\left\{S_{(1)}, ..., S_{(i)}\right\} = p_{(1)} + ... + p_{(j)} \to \min,$$

$$\begin{array}{l} \underline{p}_{(1)} \leq p_{(1)} \leq \bar{p}_{(1)}, \\ . \\ . \\ . \\ \underline{p}_{(j)} \leq p_{(j)} \leq \bar{p}_{(j)}, \\ p_{(1)} + ... + p_{(n)} = 1. \end{array}$$

Here p_0 denotes possible numeric probability of S_0. As we can see, this problem is the problem of linear programming.

Thus, the problem under consideration (choosing an optimal number of wells) is to determine an alternative with maximal value of the interval-valued utility function U:

$$U(f^*) = \max_{f_i \in A} U(f_i).$$

Determination of the maximal value of an interval-valued utility function is based on an appropriate method of comparison of intervals.

5. An application

Practical solution of the problem is as follows: let us determine the set of alternatives (the number of wells) $A = f_1, f_2, f_3, f_4 : f_1$ is an intervention in 10 wells, f_2 is an intervention in 20 wells, f_3 is an intervention in 30 wells, f_4 is intervention in 40 wells. States of nature are considered as possible geological conditions $S = S_1, S_2, S_3, S_4$ which influence outcomes of the alternatives (oil production). Due to significant uncertainty and imprecision of decision-relevant information in the considered problem, we consider interval-valued probabilities of states of nature and interval-valued outcomes of the alternatives. The decision relevant information is represented in Tab.1.

Table 1. Decision problem with interval-valued information.

	S_1	S_2	S_3	S_4
f_1	$X_{11} = [50, 60]$ $P_{11} = [0.2, 0.25]$	$X_{12} = [120, 150]$ $P_{12} = [0.3, 0.35]$	$X_{13} = [190, 200]$ $P_{13} = [0.25, 0.32]$	$X_{14} = [240, 260]$ $P_{14} = [0.1, 0.25]$
f_2	$X_{21} = [80, 100]$ $P_{21} = [0.23, 0.27]$	$X_{22} = [130, 155]$ $P_{22} = [0.24, 0.27]$	$X_{23} = [210, 230]$ $P_{23} = [0.29, 0.32]$	$X_{24} = [270, 290]$ $P_{24} = [0.14, 0.24]$
f_3	$X_{31} = [110, 120]$ $P_{31} = [0.22, 0.26]$	$X_{32} = [160, 180]$ $P_{32} = [0.27, 0.3]$	$X_{33} = [240, 260]$ $P_{33} = [0.29, 0.32]$	$X_{34} = [285, 310]$ $P_{34} = [0.12, 0.22]$
f_4	$X_{41} = [135, 150]$ $P_{41} = [0.21, 0.24]$	$X_{42} = [190, 220]$ $P_{42} = [0.29, 0.32]$	$X_{43} = [270, 300]$ $P_{43} = [0.3, 0.33]$	$X_{44} = [305, 330]$ $P_{44} = [0.11, 0.2]$

The interval probabilities used in Tab.1. come from experts' opinion and the authors' experience.

Now, we need to determine utilities of alternatives. Decision making under interval probabilities, is actually characterized by violation of independence of preferences [22]. The use of Choquet allows to account for this fact to more adequately model real-world human preferences under ambiguity. The main reason is that Choquet integral is based on non-additive measure. The latter is able to describe non-additivity of human beliefs that underlies violation of independence. The Choquet integral based utility of alternative f_1 will be determined as:

$$U(f_1) = \sum_{j=1}^{m} \left(X_{1(j)} - X_{1(j+1)} \right) \eta_{\{(1),\dots,(j)\}} =$$

$$\left(X_{1\,(1)} - X_{1\,(2)} \right) \eta_{\{(1)\}} + \left(X_{1\,(2)} - X_{1\,(3)} \right) \eta_{\{(1),\,(2)\}} +$$

$$\left(X_{1\,(3)} - X_{1\,(4)} \right) \eta_{\{(1),\,(2),\,(3)\}} + X_{1\,(4)}\ \eta_{\{(1),\,(2),\,(3),(4)\}}.$$

Here $\eta_{\{(1),\dots,(j)\}} = \eta \left\{ S_{(1)}, \dots,\ S_{(j)} \right\}$ is used for simplicity of notation.

Permutation of indices is based on the method of comparison of intervals proposed in [12]. According to this method, for two intervals $A_1 = [a_{11}, a_{12}]$ and $A_2 = [a_{21}, a_{22}]$, one has $A_1 \geq A_2$ if $a_{11} \geq a_{21}$ and $a_{12} \geq a_{22}$. Thus, given the information in Tab.1, we will obtain:

$$U(f_1) = (X_{1\,4} - X_{1\,3})\ \eta_{\{4\}} + (X_{1\,3} - X_{1\,2})\ \eta_{\{3,\,4\}} + (X_{1\,2} - X_{1\,1})\ \eta_{\{2,\,3,\,4\}} + X_{1\,1}\ \eta_{\{1,\,2,\,3,\,4\}}.$$

Utilities of the other alternatives are determined in a similar way. For calculation of utility values U we will construct measure η as the lower probability (section IV). For example, calculation of $\eta_{2,3,4}$ is as follows:

$$\eta_{\{2,3,4\}} = p_2 + p_3 + p_4 \to \min,$$

$$\begin{aligned} &0.3 \leq p_2 \leq 0.35\,, \\ &0.25 \leq p_3 \leq 0.3\,, \\ &0.1 \leq p_4 \leq 0.25\,, \\ &p_1 + ... + p_4 = 1. \end{aligned}$$

By solving this problem, we obtain $\eta_{\{2,3,4\}} = 0.75$. Similarly, we get: $\eta_{\{3,4\}} = 0.4$, $\eta_{\{4\}} = 0.1$ and it is clear that $\eta_{\{1,\,2,3,4\}} = 1$. Then we have:

$$\begin{gathered} U(f_1) = ([240,\ 260] - [190,\ 200])\ 0.1 + ([190,\ 200] - [120,\ 150])\ 0.65 + \\ ([120,\ 150] - [50,\ 60])\ 0.75 + [50,\ 60] = [115,\ 174] \end{gathered}$$

for $U(f_1)$.

Similarly, we obtain:

$$U(f_2) = [133,\ 212]\,,$$

$$U(f_3) = [169,\ 224]\,,$$

$$U(f_4) = [187,\ 268]\,.$$

Having compared the obtained interval utilities by using the method proposed in [12] we get: $U(f^*) = \max\limits_{f_i \in A} U(f_i) = U(f_4)$. The optimal alternative is f_4. Therefore, interventions carried out in 40 wells is an optimal action.

Let us compare the obtained results of decision analysis under interval information with the use of a classical approach. As a classical approach we consider Subjective Expected Utility model [21]. However, this model is developed for a precise information framework. In this model it is assumed that a DM subjectively assigns precise probabilities when dealing with decision problems under ambiguity. Therefore, to apply this model, we need to use precise values of outcomes and probabilities in the considered problem. So, we consider the following information (p, x) denote precise values of probabilities and outcomes:

Table 2. Decision problem with precise information.

	S_1	S_2	S_3	S_4
f_1	$X_{11} = 55$ $P_{11} = 0.23$	$X_{12} = 130$ $P_{12} = 0.33$	$X_{13} = 195$ $P_{13} = 0.27$	$X_{14} = 250$ $P_{14} = 0.17$
f_2	$X_{21} = 90$ $P_{21} = 0.25$	$X_{22} = 140$ $P_{22} = 0.25$	$X_{23} = 220$ $P_{23} = 0.3$	$X_{24} = 280$ $P_{24} = 0.2$
f_3	$X_{31} = 120$ $P_{31} = 0.22$	$X_{32} = 180$ $P_{32} = 0.27$	$X_{33} = 260$ $P_{33} = 0.29$	$X_{34} = 310$ $P_{34} = 0.22$
f_4	$X_{41} = 135$ $P_{41} = 0.24$	$X_{42} = 190$ $P_{42} = 0.32$	$X_{43} = 270$ $P_{43} = 0.3$	$X_{44} = 305$ $P_{44} = 0.14$

The values of Expected utility for these alternatives are as follows.

$$U(f_1) = 150.7\,,$$
$$U(f_2) = 179.5\,,$$
$$U(f_3) = 218.6\,,$$
$$U(f_4) = 216.9.$$

As one can see, the best alternative is f_3, though it is slightly better than f_4. This result differs with what we obtained when dealing with interval-valued information. Thus, disregarding of imprecision of the original information (which is always characterized by loss of information) leads to improper results. At the same time, determination of actual exact probabilities under interval information is a very time consuming and is not practically suitable.

6. Conclusion

A choice of an optimal number of oil wells is naturally characterized by imprecise information on possible outcomes and probabilities. Indeed, in such problems neither exact values of oil production nor related probabilities can be estimated due to geological, geophysical and other complex factors. In this paper, we use an interval-valued Choquet integral based utility model to solve the considered choice problem. The applied utility model allows to account for natural imprecision of decision relevant information and non-additivity of a DM's beliefs in comparison of alternatives (alternative number of wells). A real-world decision problem with four alternatives has illustrated validity and applicability of the applied interval-valued utility model.

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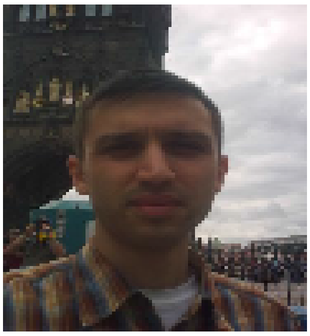

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